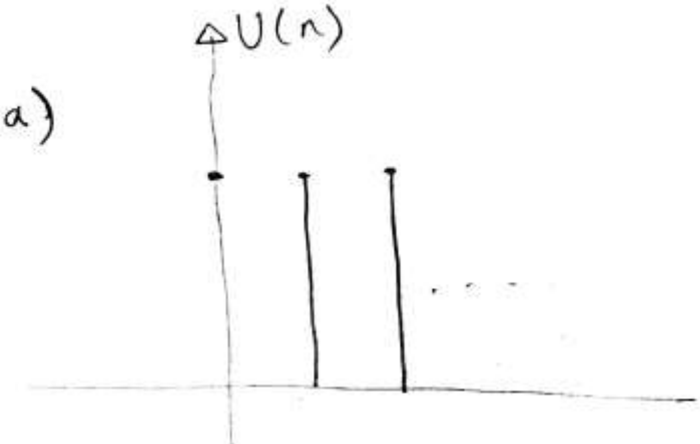
DSP : sec 3

* Properties of Convolution :-

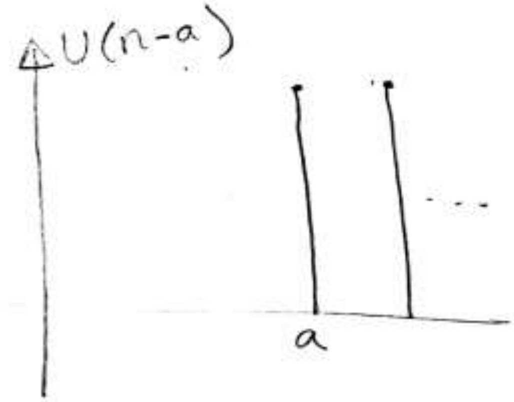
1. Commutation
2. Causality $h(n)=0 \quad , n<0$
3. stability $\sum_{-\infty}^{\infty} |h(n)| < \infty$



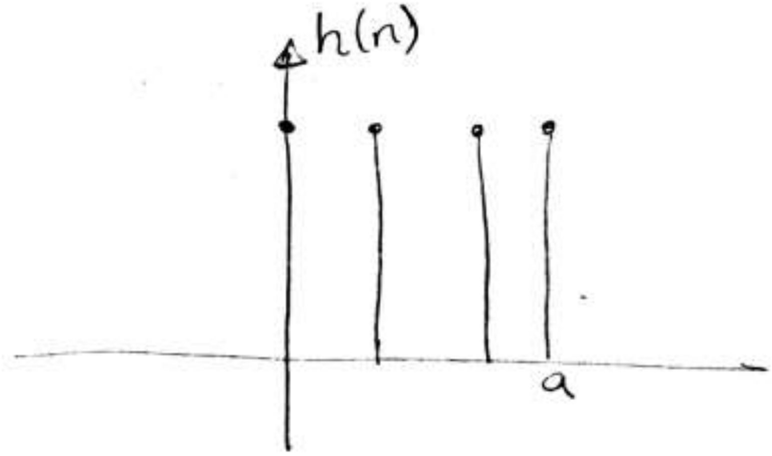
* $h(n) = U(n) - U(n-a)$

$\therefore \sum_{-\infty}^{\infty} h(n) = \sum_{n=0}^{a-1} h(n)$

$= a$



$\longrightarrow$ system is stable



Z - Transform

$$F(s) = \int_0^{\infty} f(t)\, e^{-st}\, dt \longrightarrow \text{Laplace transform (Continuous)}$$

$$F(s) = \sum_{n=0}^{\infty} f(nT)\, e^{-snT} \longrightarrow \text{Discrete}$$

assume $Z = e^{sT}$

$$\boxed{F(z) = \sum_{n=0}^{\infty} f(nT)\, z^{-n}}$$

$$F(z) = \sum_{n=0}^{\infty} f(n)\, Z^{-n}$$

$\boxed{ex}$ $x(n) = n\, e^{3n}\, u(n)$

$$x(z) = \sum_{n=0}^{\infty} n\, e^{3n}\, z^{-n}$$

$$= e^{3} z^{-1} + 2e^{6} z^{-2} + 3e^{9} z^{-3} + \cdots$$

$$= e^{3} z^{-1}\left[1 + 2e^{3} z^{-1} + 3e^{3} z^{-2} + \cdots\right]$$

$$= e^{3} z^{-1}\left[1 + 2(\underbrace{e^{3} z^{-1}}_{x}) + 3(\underbrace{e^{3} z^{-2}}_{x^2}) + \cdots\right]$$

ويكأنها $\longrightarrow$ $1 + \cancel{x}\, 2x + 3x^2 + \cdots$

$$X(z) = e^{3} z^{-1} \frac{1}{(1 - e^{3} z^{-1})^{2}}$$

Note that

* $1 + x + x^{2} + x^{3} + \cdots \longrightarrow \dfrac{1}{1-x}$

* $1 + 2x + 3x^{2} + \cdots \longrightarrow \dfrac{1}{(1-x)^{2}}$

* $1 - 2x + 3x^{2} - \cdots \longrightarrow \dfrac{1}{(1+x)^{2}}$

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another solution

$$X(z) = e^{3} z^{-1} + 2 e^{6} z^{-2} + 3 e^{9} z^{-3} \cdots$$

$$e^{3} z^{-1} X(z) = e^{6} z^{-2} + 2 e^{9} z^{-3} \cdots$$

$$X(z) - e^{3} z^{-1} X(z) = e^{3} z^{-1} + e^{6} z^{-2} + e^{9} z^{-3} \cdots$$

$$X(z)\left[1 - e^{3} z^{-1}\right] = z^{-1} e^{3} \left[1 + e^{3} z^{-1} + e^{6} z^{-2} + \cdots\right]$$

$$= \frac{e^{3} z^{-1}}{1 - e^{3} z^{-1}}$$

$$X(z) = \frac{e^{3} z^{-1}}{(1 - e^{3} z^{-1})^{2}}$$

* $x(n) = (-1)^n \, u(n)$

$$X(z) = \sum_{n=0}^{\infty} (-1)^n z^{-n}$$

$$= 1 + (-1) z^{-1} + (-1)^2 z^{-2} + (-1)^3 z^{-3} \cdots$$

$$= 1 - z^{-1} + z^{-2} - z^{-3} \cdots$$

$$= \frac{1}{1 + z^{-1}}$$

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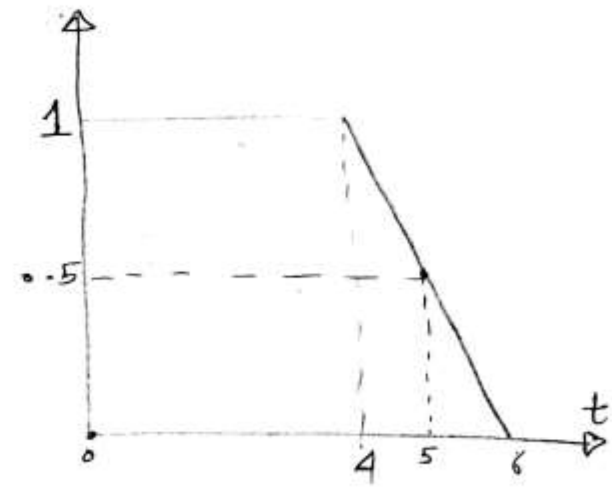
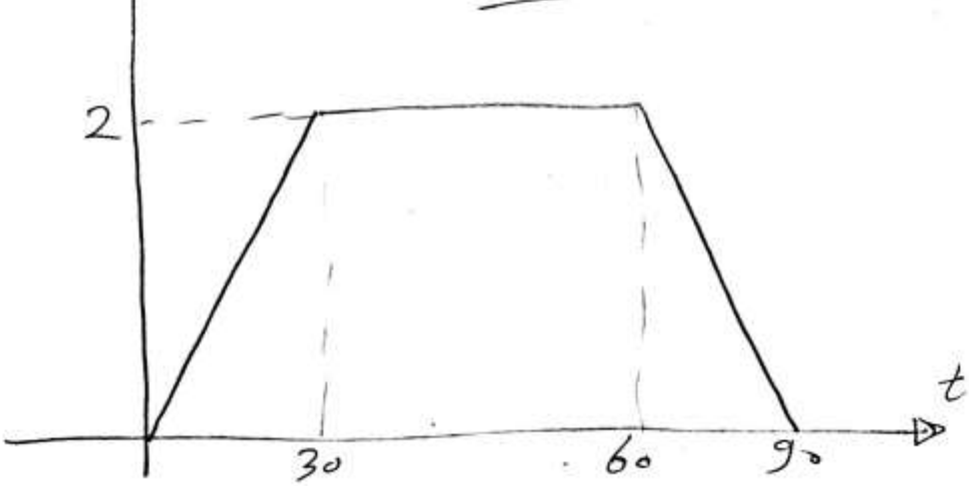
$x(n) = \{1, 1, 1, 1, 1, 0.5, 0\}$

$$X(z) = \sum_{0}^{\infty} x(n) \, z^{-n}$$

$$= 1 + z^{-1} + z^{-2} + z^{-3} + \cdots$$



Report



Properties

* $Z[a\,x(n)] = a\,X(z)$

* $Z[x_1(n) \pm x_2(n)] = X_1(z) \pm X_2(z)$

* $x_1(n) * x_2(n) \longrightarrow X_1(z)\,X_2(z)$

* $Z[e^{\pm an} x(n)] \longrightarrow X(z)\Big|_{z = z e^{\mp an}}$

* $Z[n\,x(n)] = -z \frac{d}{dz} X(z)$

* $Z[a^n x(n)] = X(z)\Big|_{z = \frac{z}{a}}$

* $Z(x(n-m)) = z^{-m} X(z)$

$\boxed{ex}$ $Z\left((-1)^n u(n)\right)$

$\because Z[u(n)] = \frac{z}{z-1}$

$$Z[(-1)^n u(n)] = \frac{z}{z-1}\Big|_{z = \frac{z}{-1}} = \frac{-z}{-z-1}$$

$$= \frac{z}{z+1} = \frac{1}{1+z^{-1}}$$

$$* \quad x(n) = (0.5)^n + (0.25)^n + 3\,\underbrace{\delta(n-2)}_{\text{shift Properity.}}$$

$$X(z) = \frac{z}{z-0.5} + \frac{z}{z-0.25} + 3z^{-2}(1)$$

$$* \quad x(n) = \underbrace{(-1)^n \sin(wn)} + \underbrace{e^{-2n}\cos wn}$$

$$Z[\sin(wn)]\Big|_{z=\frac{z}{-1}} \qquad Z[\cos(wn)]\Big|_{z=ze^{2n}}$$

$$* \quad X(s) = \frac{10}{(s+1)(s+2)}$$

$$= \frac{a}{s+1} + \frac{b}{s+2} \Rightarrow x(t) = ae^{-t} + be^{-2t}$$

$t \rightarrow nT$ (and $T=1$) $\therefore x(n) = ae^{-n} + be^{-2n}$

$$\therefore X(z) =$$

$* \; y(n) = x_1(n) * x_2(n)$

$$x_1(n) = (0.5)^n \, U(n-1)$$

$$x_2(n) = \delta(n) \, e^{2n}$$

$$X_1(z) = Z\left[U(n-1)\right]\Big|_{z = \frac{z}{0.5}}$$

$$= z^{-1} \, Z\left[U(n)\right]\Big|_{z = \frac{z}{0.5}}$$

$$= \frac{z^{-1} z}{z-1}\Big|_{z = \frac{z}{0.5}} = \frac{1}{2z-1}$$

$$= 2z$$

$$X_2(z) = \text{``}$$

$$Y(z) = X_1(z) * X_2(z)$$